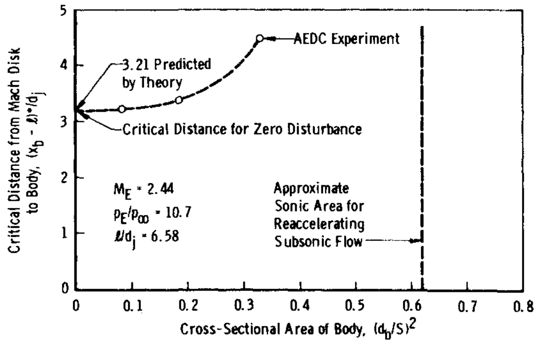

Fig. 6 Position of critical station downstream of Mach disk.[3]

Test Facility of the AEDC offers three additional data points for comparison, two of which were obtained with a Mach 2.44 nozzle and are presented in Fig. 2. At a p_E/p_∞ of 5.75, an l/d_j of 4.96 was obtained as against the theoretical prediction of 4.94. Again at a p_E/p_∞ of 10.7, an l/d_j of 6.58 was recorded compared with the prediction of 6.53.

A third experiment was devised by Peters to determine the location of the sonic region. In that experiment, shown schematically in Fig. 5, blunt cylindrical bodies of various diameters were translated upstream towards the Mach disk of the Mach 2.44 nozzle. At some critical body position, $(x_b - l)^*$, observation with shadowgraph showed the location of the Mach disk to be affected by the particular body. By extrapolation of the experimental critical distances to zero body diameter (i.e., to zero disturbance), the location of the critical station was established. The results presented in Fig. 6 for $p_E/p_\infty = 10.7$ show that the critical station is located approximately 3.2 nozzle diameters downstream of the Mach disk. The theory predicts a corresponding distance of 3.21 diameters. The extremely close correlation is of course fortuitous, but even accepting error in the experimental procedure, the agreement is quite good.

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Specific Heat of X14 Propellant

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Nomenclature

$\bar{C}_p$ = specific heat of sample
$\bar{C}_{p_{std}}$ = specific heat of sapphire standard
m_s = mass of sample
m_{std} = mass of sapphire standard
ΔY_s = magnitude of thermal lag for sample
ΔY_{std} = magnitude of thermal lag for sapphire standard

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Introduction

RESEARCH is in progress in our laboratory to measure and eventually model the temperature sensitivity of solid propellant combustion. Current experiments are being conducted with X14, a solid, nitroglycerin-nitrocellulose double-base propellant. The specific heat of the condensed phase is one physical parameter needed to compare experiments with proposed models. In the past the specific heat was calculated by summing the atomic contributions to the heat capacity, and then dividing by the propellant's molecular weight. Measured heat capacities are not available for NC-NG double-base propellants; instead the calculated specific heat must be used over the temperature range of interest. Kirby and Suh[1] calculated the condensed phase contribution to the over-all heat of reaction of M2 propellant using a constant calculated value of 0.27 cal/g-°C for the specific heat of solid M2 over a 125°C range. For X14 propellant, not even a calculated specific heat is available, so it was decided to measure the specific heat with a differential scanning calorimeter. This will provide accurate specific heat data for the X14 temperature sensitivity experiments, as well as a measured specific heat to compare with the calculated specific heats for NC-NG double-base propellants. Since the differential scanning calorimeter measures specific heat continuously with changing temperature, the assumption that the propellant's specific is temperature invariant may be tested too.

Experimental

Specific heats were measured with a thermal analyzer (DuPont model 990) equipped with the differential scanning calorimeter (DSC) module. This DSC cell makes use of a constantan disk with two raised platforms for the aluminum sample and reference pans. The constantan disk serves as the primary means of heat transfer to the sample and reference pans. The thermal lag between these is proportional to the mass and specific heat of the sample and reference systems for a given heating rate. Specific heats are determined by comparing the thermal lag under "blank" and "sample" conditions. A blank run consists of two empty aluminum sample and reference pans; the sample was rerun under identical conditions with the X14 propellant added to the sample pan. Baxter[2] reports that specific heats of polymeric materials may be determined to within 5% of existing values with this type DSC cell. The DSC cell has a calorimetric sensitivity of 0.05 mcal/sec-in.

Three disks of X14 propellant were cut from the center of a block of the same propellant lot used in the temperature sensitivity experiments. Specific heats were measured from 10°C to 70°C, since a heating curve of weight vs temperature of X14 propellant obtained with a thermogravimetric analyzer

Table 1 Specific heat of X14 propellant samples[a]

Temp, °K	$\bar{C}_p$ X14[b]	$\bar{C}_p$ X14[c]	$\bar{C}_p$ X14[d]
283	0.307	0.306	0.299
288	0.309	0.309	0.302
293	0.315	0.315	0.307
298	0.317	0.318	0.310
303	0.322	0.323	0.313
308	0.324	0.326	0.317
313	0.327	0.330	0.320
323	0.333	0.338	0.326
333	0.340	0.452	0.332
343	0.345	0.347	0.337

[a] $\bar{C}_p$ in units of cal/g-K.
[b] 22.80 mg sample.
[c] 49.14 mg sample.
[d] 53.58 mg sample.

Table 2 Specific heat of X14 propellant and 95% confidence intervals

T, °K	$\bar{C}_p$, cal/g-K	$\bar{C}_p$, J/kg-K
283	0.304 ± 0.013	1270 ± 50
285	0.307 ± 0.012	1280 ± 50
293	0.312 ± 0.014	1310 ± 60
298	0.315 ± 0.013	1320 ± 50
303	0.319 ± 0.017	1330 ± 70
308	0.323 ± 0.015	1350 ± 60
313	0.236 ± 0.016	1360 ± 70
323	0.332 ± 0.018	1390 ± 80
333	0.338 ± 0.016	1410 ± 70
343	0.343 ± 0.016	1440 ± 70

(DuPont 951) showed a detectable weight loss commencing at 75°C. Kirby and Suh[3] observed a similar phenomenon during heating of another double-base propellant which they attributed to volatilization of the nitroglycerin. The X14 propellant was reweighed at the end of each specific heat determination and showed that no weight change had occurred. All weighings were made with a Cahn RG Electrobalance and all the DSC runs were made in air. The DSC cell is calibrated by running a sample with known specific heat. A sapphire disk in the DSC cell accessory kit is provided for this purpose.

Specific Heat Determination

DSC curves of three X14 propellant samples were run from 10°C to 70°C, along with two calibration runs with the sapphire disk. Specific heat at ten temperatures was determined using the following equation

$$\bar{C}_p = \bar{C}_{p_{std}} x (m_{std}/\Delta Y_{std}) x (\Delta Y_s/m_s)$$

The results for the three X14 samples and the specific heats used for the sapphire standard are given in Table 1. Table 2 lists the mean specific heat for X14 propellant along with the 95% confidence intervals at each temperature.

Kelley's[4] three-term equation given below was used to algebraically represent the specific heat as a function of temperature.

$$\bar{C}_p = a + bT - cT^{-2}$$

An examination of the specific heats in Table 2 suggested that only the first two terms are necessary to fit the data, so the specific heats in Table 2 were fitted to the previous expression with a nonlinear least squares program[5] with c set equal to zero. The best-fit values of a and b obtained from this calculation are as follows:

$$a = 0.118 \text{ cal/g-K} = 494 \text{ J/kg-K}$$
$$b = 6.60 \cdot 10^{-4} \text{ cal/g-K}^2 = 2.76 \text{ J/kg-K}$$

Table 3 lists the experimental specific heats and the specific heats generated using the best-fit values. In all cases the deviation is well within the 95% confidence intervals, so the two-term equation is deemed adequate to represent the specific heat of X14 from 283-343K.

Table 3 Comparison of specific heats generated with best-fits, values of a and b with experimental values

T, °K	$\bar{C}_p$, (expt), cal/g-K	$\bar{C}_p$, (best-fit), cal/g-K
283	0.304	0.305
288	0.307	0.308
293	0.312	0.312
298	0.314	0.315
303	0.319	0.318
308	0.323	0.322
313	0.326	0.325
323	0.332	0.332
323	0.338	0.338
343	0.343	0.345

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Integral Solution of the Turbulent Energy Equation

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Nomenclature

A, b	= constants of law of the wall
a_1, G, L	= Bradshaw turbulent parameters
a_4	= parameter of shear distribution
u, v	= velocity components
$x, yz \equiv y/\delta$	= coordinates
τ	= shear
δ, δ^*	= thickness and displacement thickness of boundary layer
c_f	= skin-friction coefficient
π	= Coles wake parameter

THE turbulent energy equation in the form proposed by Bradshaw, Ferris and Atwell,[1] namely

$$u\frac{\partial}{\partial x}\left(\frac{\tau}{2a_1\rho}\right) + v\frac{\partial}{\partial y}\left(\frac{\tau}{2a_1\rho}\right) - \frac{\tau}{\rho}\frac{\partial u}{\partial y} + \left(\frac{\tau_{\max}}{\rho}\right)^{1/2}\frac{\partial}{\partial y}\left(G\frac{\tau}{\rho}\right) + \frac{(\tau/\delta)^{3/2}}{L} = 0 \tag{1}$$

lends itself to a simple integral solution, once a representation of shear and velocity profiles through the boundary layer is postulated.

First it is noted that in the vicinity of the wall, production and dissipation are balanced [3rd and 5th term of Eq. (1)]. Then, Eq. (1) can be integrated across the boundary layer from a location y_1 close to the wall to δ, to yield

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